Blind Calibration for Phase Shifts in Compressive Systems

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***Abstract*—We consider a *blind* calibration problem in a compressed sensing measurement system in which each sensor introduces an unknown phase shift to be determined. We show that this problem can be approached similarly to the problem of phase retrieval from quadratic measurements. Furthermore, when dealing with measurements generated from *multiple* unknown (but sparse) signals, we extend the approach for phase retrieval to solve the calibration problem in order to recover the signals *jointly* along with the phase shift parameters. The proposed methods are shown to have significantly better recovery performance than individual recovery of the input signals when the number of input signals are sufficiently large.**

I. INTRODUCTION

We consider a compressive measurement system that is perturbed by unknown complex gains at each sensor i for which there are multiple K-sparse input signals, $\mathbf{x}_l \in \mathbb{C}^N, \quad l = 1 \dots L$, applied to the system such that

$$y_{i,l} = d_i e^{j\theta_i} \mathbf{m}_i' \mathbf{x}_l \qquad i = 1 \dots M,\ \theta_i \in [0, 2\pi),\ d_i \in \mathbb{R}^+ \qquad (1)$$
$$\mathbf{m}_i \in \mathbb{C}^N \text{ : known, } .' \text{ : Conj. Transpose}$$

A special case of this problem with θ_i known has been studied in [1]. In this work, we investigate an alternative special case for this problem, which we call the *Phase Calibration*, where the gain magnitudes, d_i, are known, and calibration consists in determining the unknown phase shifts for each sensor, θ_i. Hence $d_i \mathbf{m}_i$ is simply replaced with $\mathbf{m}_i$ for the rest of the discussions. We focus only on the noiseless case for the sake of simplicity.

II. PHASE CALIBRATION - AN EXTENSION TO PHASE RETRIEVAL

Let us define the cross measurements, $g_{i,k,l}$ as

$$g_{i,k,l} \triangleq y_{i,k} y_{i,l}' = e^{j\theta_i} \mathbf{m}_i' \mathbf{x}_k \mathbf{x}_l' \mathbf{m}_i e^{-j\theta_i} = \mathbf{m}_i' \mathbf{X}_{k,l} \mathbf{m}_i \qquad (2)$$
$$i = 1 \dots M,\ k, l = 1 \dots L,\ \mathbf{X}_{k,l} \triangleq \mathbf{x}_k \mathbf{x}_l' \in \mathbb{C}^{N \times N}$$

We can also define the joint signal matrix $\mathbf{X} \in \mathbb{C}^{LN \times LN}$

$$\mathbf{X} \triangleq \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_L \end{bmatrix} \left[\mathbf{x}_1' \cdots \mathbf{x}_L'\right] = \mathbf{x}\mathbf{x}' = \begin{bmatrix} \mathbf{X}_{1,1} & \cdots & \mathbf{X}_{1,L} \\ \vdots & \ddots & \vdots \\ \mathbf{X}_{L,1} & \cdots & \mathbf{X}_{L,L} \end{bmatrix} \qquad (3)$$

which is rank-one, positive semi-definite and sparse when the input signals, $\mathbf{x}_l$, are sparse. Therefore we propose to recover the joint matrix $\mathbf{X}$ with the semi-definite program

$$\begin{aligned} \mathbf{X}^* = \arg\min_{\mathbf{Z}} \quad & \mathrm{trace}(\mathbf{Z}) + \lambda \|\mathbf{Z}\|_1 & & (4) \\ \text{subject to} \quad & g_{i,k,l} = \mathbf{m}_i' \mathbf{Z}_{k,l} \mathbf{m}_i & k, l = 1 \dots L & \\ & \mathbf{Z} \succcurlyeq 0 & i = 1 \dots M & \end{aligned}$$

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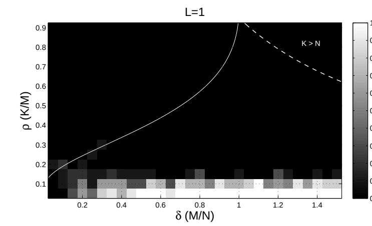
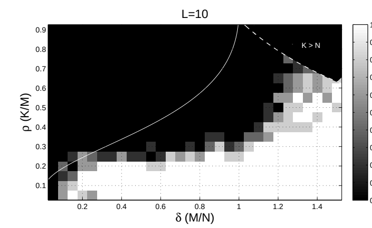


(a) Compressive Phase Retrieval via Lifting (CPRL) [2] (eqv. to **Joint** opt., $L = 1$)



(b) **Joint** matrix opt., $L = 10$

Fig. 1: The probability of perfect recovery for $N = 100$ with respect to $\delta \triangleq M/N$ and $\rho \triangleq K/M$. The solid line indicates the Donoho-Tanner phase transition curve for fully calibrated compressed sensing recovery. The dashed line indicates the boundary to the region where $K > N$.

which is an extension to phase retrieval algorithms proposed in [2] through joint processing of input signals. The estimated signal, $\mathbf{x}^*$ (and therefore $\mathbf{x}_1^*, \dots \mathbf{x}_L^*$) is defined up to a global phase since $\mathbf{X}^* = \mathbf{x}^* \mathbf{x}^{*\prime}$. The phases θ_i can be recovered given $y_{i,l}$ and $\mathbf{x}^*$. Figure 1 presents the probability of recovery of the input signals from uncalibrated measurements for our joint recovery method (Figure 1b) and the individual recovery method by Ohlsson *et al.* [2] (Figure 1a). These diagrams show that the joint recovery method outperforms the individual one under wide range of conditions when L is high enough.

The talk will present further performance analysis of the proposed method for phase calibration, and discuss alternative recovery methods with simplified complexity retaining the performance improvements through joint processing. The approach combining the introduced methods for the general problem with unknown d_i and θ_i will also be discussed.

REFERENCES

[1] Rémi Gribonval, Gilles Chardon, and Laurent Daudet, “Blind calibration for compressed sensing by convex optimization,” in *Acoustics Speech and Signal Processing (ICASSP), 2012 IEEE International Conference on*, 2012, pp. 2713–2716.

[2] Henrik Ohlsson, Allen Y Yang, Roy Dong, and Shankar S. Sastry, “Compressive phase retrieval from squared output measurements via semidefinite programming,” *arXiv preprint arXiv:1111.6323*, 2011.